Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 7, 2004*
Accepted: *November 10, 2004*

Non-critical, near extremal AdS_6 background as a holographic laboratory of four dimensional YM theory

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Abstract: We study certain properties of the low energy regime of a theory which resembles four dimensional YM theory in the framework of a non-critical holographic gravity dual. We use for the latter the near extremal AdS_6 non-critical SUGRA. We extract the glueball spectra that associates with the fluctuations of the dilaton, one form and the graviton and compare the results to those of the critical near extremal $D4$ model and lattice simulations. We show an area law behavior for the Wilson loop and screening for the 't Hooft loop. The Luscher term is found to be $-\frac{3}{24}\frac{\pi}{L}$. We derive the Regge trajectories of glueballs associated with the spinning folded string configurations.

Keywords: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence, Lattice QCD.

JHEP11(2004)026

Contents

1. Introduction and summary

Ten dimensional type-IIB superstring theory on $AdS_5 \times S^5$ is holographically equivalent to four dimensional $\mathcal{N} = 4$ super Yang Mills theory. The S^5 transverse space with its SO(6) isometry is required to dualize the R symmetry of the boundary gauge theory. Using this logic the superstring dual of $\mathcal{N} = 1$ SYM should admit an S^1 as a transverse dimension and the dual of the non-supersymmetric YM theory should be five dimensional with no transverse space. The fifth dimension plays the role of the re-normalization scale of the dual gauge theory.

Critical ten dimensional string (SUGRA) theories which are the anti-holographic duals of confining gauge theories with $\mathcal{N} = 1$ supersymmetry are characterized by a KK sector of states which are of the same mass scale as the hadronic states. There is no apparent way to disentangle these KK states from the hadronic states. This situation shows up for instance in the KS [1] and MN [2] models.

The KK modes combined with the argument about the R symmetry serves as a motivation to study non-critical string theories, as candidates for the string of QCD. This idea was originally introduced in [3] where a proposal of a dual of pure YM in terms of a 5d non-critical gravity background was made. Following this paper there were several attempts to find solutions of the non-critical effective action that are adequate as duals of gauge theories [4]–[14].

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Recently, in [15] we investigated the supergravity equations of motion associated with non-critical ($d > 1$) type-II string theories that incorporate RR forms. Several classes of solutions were derived. In particular we found analytic backgrounds with a structure of $AdS_{p+2} \times S^k$ and numerical solutions that asymptote a linear dilaton with a topology of $R^{1,d-3} \times R \times S^1$. Unfortunately, for all these solutions the curvature in string units is proportional to $c = 10 - d$ and it cannot be reduced by taking a large-N limit like in the critical case. This means that the supergravity approximation is not really valid. We conjectured, however, that the higher order corrections will modify the radii, while leaving the geometrical structure of the background unchanged. We also presented the AdS black hole backgrounds associated with the $AdS_{p+2} \times S^k$ solutions. In [16] the elevation of the SUGRA non-critical background into sigma models was discussed. Models with AdS target spaces which have κ-symmetry and are completely integrable were derived. Another important development in this program of deriving the string of QCD has been made in [17] where the $AdS_5 \times S^1$ was derived from a non-critical SUGRA action that includes a term associated with a space filling flavor brane (see also [18] for the recent progress).

The goal of this paper is to go one step forward and extract gauge dynamical properties of confining theories from a non-critical SUGRA laboratory. In [15] two classes of SUGRA backgrounds were shown to be duals of confining gauge theories: (i) near extremal AdS backgrounds (ii) backgrounds that asymptote a linear dilaton background. Since the latter were numerical solutions we choose to use the former option, namely a thermal AdS background. To describe a four dimensional confining gauge theory we use the near extremal AdS_6 background. The AdS_6 model, which is a member of the family of solutions mentioned above, includes a zero form field strength that may be associated with a $D4$ brane in a similar way to the $D8$ brane of the critical type-IIA superstring theory. Near extremality, or the incorporation of a black hole, is achieved by compactifying one of the $D4$ world-sheet coordinates on a circle and imposing anti-periodic boundary conditions. Some preliminary calculations have already been made in [15]. In this paper we consider both SUGRA as well as semi-classical string computations. The first kind includes the extraction of the glueball spectra associated with the fluctuations of the dilaton, the graviton and the one form. Wilson loops, 't Hooft loops and spinning folded closed string are among the second kind.

An important question that is addressed in this paper is whether the results about the gauge dynamics that one derives from the non-critical models differ from those one extracts from the critical ones and in what ways. Here is brief summary of the answer to this question.

- As advertised above, in comparison with the critical near extremal $D4$ brane model, the non-critical model lacks the KK modes associated with the transverse S^4. And, thus, in our case the corrections to everything we compute can be large.

- The gauge theory dual of the non-critical SUGRA, is a theory in the large-N limit. However, unlike the usual limit of the AdS/CFT correspondence, here 't Hooft parameter is of order one $g^2 N \sim 1$ and not very large.

- Though the derivation of the glueball spectrum is similar to the one performed in the critical case there are certain differences in the resulting glueball spectra. For instance, in the critical supergravity approach there is a degeneracy between the 2^{++}, 1^{++} and 0^{++} glueball states. This degeneracy is not present in our results. On the other hand, we find that $M(2^{++})/M(0^{-+}) = 1$ is similar to the critical supergravity result $M(2^{++})/M(0^{-+}) \approx 1.2$ and the lattice result $M(2^{++})/M(0^{-+}) \approx 1.08$.

- The classical string configurations that correspond to the Wilson loop and the 't Hooft loop admits the same behavior in the ten dimensional and six dimensional string models, namely, area law behavior for the former and screening for the latter. However the corresponding string tension behaves like $T_s \sim \sqrt{g^2 N} T^2$ and $\sim T^2$ respectively, where T, the "temperature" is the inverse of the circumference of the compactified S^1. A similar difference occurs also for the tension of the folded spinning closed strings that admit a Regge trajectory behavior.

- The quantum fluctuations around the above classical configurations differ between the critical and non-critical models simply due to the fact that the number of bosonic (and presumably also fermionic) directions is different.

  For instance the Luscher term that measures the quantum deviation from the linear potential was shown to be $-\frac{7}{24}\frac{\pi}{L}$ in the critical case where L is the separation distance between the corresponding quark and anti-quark. Assuming that the fermionic fluctuations are massive due to the coupling to the RR fluxes, as is the case in the critical model, the bosonic contributions take the form of $-\frac{3}{24}\frac{\pi}{L}$, which is closer to the result found in lattice simulations.

- The KK modes of the critical string model take part as intermediate states in amplitudes of hadronic states. Intermediate states in non-critical models of the type we consider here do not include these KK modes. For instance it was shown in [19] that the dominant mode in the correlator of two Wilson loops is a KK mode in the critical model. In the non-critical case it is replaced by a glueball mode.

The organization of the paper is as follows. In section 2 we present the non-critical supergravity solution. Section 3 is devoted to the calculation of the glueball spectrum in the gauge theory in the context of non-critical supergravity. We find an expression for the WKB approximation to the spectrum, then compare it with numerical results for the lowest states. We also compare our results to the lattice YM_4 results. Although the lattice calculations and the supergravity calculations are valid in different regions (weak and strong coupling respectively) we find that there is a qualitative agreement between the results. In section 4 we give the stringy description of the $4d$ Wilson line. As expected the background satisfies the condition for the area law behavior. We also compute the classical energy of the Wilson line. In section 5 we discuss the 't Hooft line. Similarly to the critical case the calculations in the non-critical background show that there is a screening of magnetic charge in the gauge theory. In section 6 we analyze the equation of motion of the string sigma model in the given back-

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ground. We find a remarkably simple semi-classical dispersion relation for a closed rotating string configuration. This relation describes a Regge trajectory in the dual gauge theory.

2. General setting

The non-critical AdS black hole solution presented in [15] has the following form:

$$l_s^{-2}ds^2 = \left(\frac{u}{R_{AdS}}\right)^2 \left[f(u)d\theta^2 - dt^2 + \sum_{i=1}^{p-1} dx_i^2\right] + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{f(u)} + R_{S^k}^2 d\Omega_k^2\,, \tag{2.1}$$

where $f(u) = \left(1 - \left(\frac{u_\Lambda}{u}\right)^{p+1}\right)$ is the thermal factor. We denote here the location of the horizon by u_Λ to indicate the fact that it determines the scale Λ_{QCD} in the dual gauge theory. Other authors denote it as u_0 or u_T or u_{KK}. The radii appearing in the metric are:

$$R_{AdS} = \left(\frac{(p+1)(p+2-k)}{c}\right)^{1/2} \qquad \text{and} \qquad R_{S^k} = \left(\frac{(p+2-k)(k-1)}{c}\right)^{1/2}. \tag{2.2}$$

For $(p,k) = (3,5)$ and $(p,k) = (5,4)$ this metric was considered in [20, 21] (for the radii of the critical case), where the calculation of the glueball spectra in YM_3 and YM_4 was performed using critical $d = 10$ supergravity. In this paper we will analyze the $(p,k) = (4,0)$ case in the context of non-critical supergravity. This will provide an alternative "non-critical" description of YM_4.

Apart from the metric (2.1) the background includes the constant dilaton:

$$e^{2\phi_0} = \frac{2c}{p+2}\frac{1}{Q^2} \qquad \text{with } c = 10 - d = 8 - p - k\,. \tag{2.3}$$

Here Q is the RR flux. The coordinate θ has to be periodic in order to avoid a conical singularity at $u = u_\Lambda$. The circle parameterized by θ shrinks smoothly to zero if the period is chosen to be:

$$\beta = \frac{4\pi R_{AdS}^2}{(p+1)u_\Lambda}\,. \tag{2.4}$$

3. The glueball spectra

According to the AdS/CFT correspondence [22] in order to determine the glue-ball mass spectrum we must solve the linearized supergravity equations of motion in the background (2.1). For calculation of the glue-ball spectra in the framework of the critical supergravity see [21, 23, 20],[24]–[28]. In $d = 6$ the supergravity excitation we are interested in are those of the dilaton, the metric and the RR one-form appearing in the non-critical action. We should note that the RR 3-form in $d = 6$ is not an independent field since its 4-form field strength is Hodge dual to the 2-form field strength of the RR 1-form. There is also a possibility to add the NS-NS 2-form field perturbation, but we will not do it in this paper.

It appears that perturbing the equations of motion in the string frame leads to a complicated mixing between the dilaton and the metric modes. We therefore will switch to the Einstein frame, where the action takes the following form:

$$S = \int d^d x \sqrt{g} \left(\mathcal{R} - \frac{4}{d-2} (\partial\phi)^2 + \frac{c}{\alpha'} e^{\frac{4}{d-2}\phi} \right) - \frac{1}{2} \sum_{l=2,6} \int e^{\frac{d-2l}{d-2}\phi} F_l \wedge \star F_l \,. \tag{3.1}$$

In the following we will perform the calculation for general p, while substituting the relevant value $p = 4$ ($d = 6$) only in the final results. The background solution of (3.1) involves the AdS black hole metric (2.1), the constant dilaton $\phi^{(0)}$ and the RR $(p+2)$-form, which plays the role of the cosmological constant as we pointed out above. We write the perturbed fields as $g_{MN} = g^{(0)}_{MN} + h_{MN}$ and $\phi = \phi^{(0)} + \bar{\phi}$ and arrive at the following set of linearized equations of motion for the metric, the dilaton and the RR 1-form:

$$\nabla_M \nabla_N h^L_L + \nabla^2 h_{MN} - 2\nabla^L \nabla_{(M} h_{LN)} = \frac{2(p+1)}{R^2_{AdS}} h_{MN} \,, \tag{3.2}$$

$$\nabla^2 \bar{\phi} = \frac{(p+1)(p+2)}{R^2_{AdS}} \bar{\phi} \,, \tag{3.3}$$

$$\nabla^M F^{(2)}_{MN} = 0 \,, \tag{3.4}$$

where in the Einstein frame $R^2_{AdS} = \frac{1}{c}(p+1)(p+2)e^{-\frac{4}{p}\phi^{(0)}}$. We see that the metric satisfies the linearized version of the usual Einstein equation in $p+2$ dimensions with a negative cosmological constant. On the other hand the dilaton equation (3.3) differs from the laplacian equation $\nabla^2 \bar{\phi} = 0$ one obtains in the critical $AdS_5 \times S^5$ case. The term on the r.h.s. of (3.3) appears due to the non-critical term in the action (3.1). Remarkably this equation is similar to the dilaton equation in the critical dimension, which includes the contribution of the non-zero KK modes on the compact transversal space.

The WKB approximation of the graviton spectrum in the AdS black hole background was thoroughly analyzed by [24] for arbitrary p. Generally one may consider various polarization of the graviton. This results in states that correspond to the 2^{++}, 1^{-+} and 0^{++} glueball spectra in the dual gauge theory. In this section we will quote the results of the WKB calculation in the graviton case and will compare it to numerical results. We will also analyze the dilaton and RR 1-form equations. Exactly as in the 10d critical case we can assign the correct parity and charge conjugation eigenvalues to the corresponding glueball states by considering the coupling between the supergravity fields and the boundary gauge theory. This coupling can be determined from a DBI action plus a WZ term for a single D4-brane in the given background. For instance, from the WZ part one finds the RR field $A_{(1)}$ with a leg along the compact θ-direction couples to the gauge field $F_{\mu\nu}$ through the term $\epsilon^{ijkl} A_\theta F_{ij} F_{kl}$. It means that this term $(P, C) = (-, +)$ and hence $A_\theta \to 0^{-+}$. Similarly, for the RR field polarized along the non-compact world volume directions we obtain $A_\mu \to 1^{++}$. We also have $\phi \to 0^{++}$, since the dilaton couples to to the operator ϕF^2.

3.1 The dilaton and the RR 1-form spectra

We start by introducing the ansatz for the the dilaton field $\bar{\phi} = b(u)e^{ikx}$, where $b(u)$ depends only on the radial coordinate and k_μ is a p-vector along the Minkowski part of the metric.

We then have $M^2 = -k^2$ as the Lorentz invariant mass-squared of the dual operator. We will not discuss here the KK modes related to solutions with a non-trivial dependence on the compact coordinate θ. Plugging this ansatz into the dilaton equation (3.3) and using the explicit form of the metric (2.1) for $k = 0$ we obtain a 2nd order differential equation for $b(u)$:

$$\partial_u \left(\left(u^{p+2} - u \right) \partial_u b(u) \right) + \left(M^2 R_{AdS}^4 u^{p-2} - (p+1)(p+2) u^p \right) b(u) = 0 \,. \tag{3.5}$$

Here the last term appears due to the non-laplacian part in the equations of motion. In order to put the equation into a Schröedinger like form we follow [24] and re-define the wave function according to $b(u) = \gamma(u)\xi(u)$ with:

$$\gamma(u) \equiv \sqrt{\frac{u - u_\Lambda}{u \left(u^{p+1} - u_\Lambda^{p+1} \right)}} \tag{3.6}$$

and introduce a new radial coordinate defined by $u = u_\Lambda \left(1 + e^y\right)$. Now the equation (3.5) reduces to:

$$-\xi''(y) + V(y)\xi(y) = 0 \,. \tag{3.7}$$

where the effective potential takes the following form:

$$\begin{aligned} V(y) \; &= \frac{1}{4} + \frac{e^{2y} \left(p(p+2)(1+e^y)^{2(p+1)} - 2p(p+2)(1+e^y)^{p+1} - 1 \right)}{4(1+e^y)^2((1+e^y)^{p+1}-1)^2} - \\ &- \left(\left(\frac{M R_{AdS}^2}{u_\Lambda} \right)^2 - (p+1)(p+2)(1+e^y)^2 \right) \frac{e^{2y}(1+e^y)^{p-3}}{(1+e^y)^{p+1} - 1} \,. \end{aligned} \tag{3.8}$$

Although the analytic expression (3.8) is very complicated, the potential still has a relatively simple shape as it shown on figure 1.

Before proceeding, note that for $u \gg u_\Lambda$ (at large y) one has $\gamma(u) \approx \left(\frac{u}{R}\right)^{-(p+1)/2}$ and therefore for any function $\xi(y)$, which goes to zero at $y \to \infty$, the associated wave function $b(u)$ is normalizable with respect to the AdS_{p+2} metric measure.

Given the explicit form of the potential the mass parameter may be fixed be requiring that the Schröedinger equation (3.7) produces a bound state at zero energy. Despite the complicated form of the potential, it is, however, clear that there are no tachyonic (or $M^2 = 0$) modes in this case, since for $M^2 < 0$ the potential $V(y)$ is positive everywhere. Hence the corresponding operator in the gauge theory has a mass gap.

We will analyze the spectrum of the excitations applying the WKB approximation following the method developed in [26]. This approximation is valid for large M^2, where the potential well is sufficiently deep. The asymptotic behavior of the potential (3.8) is given by:

$$\begin{aligned} V(y \to \infty) &\approx \frac{1}{4}(p+1)(5p+9) - \frac{1}{2}((p+1)(5p+9)-1)e^{-y} + \\ &\quad + \left(\frac{3}{4}((p+1)(5p+9)-1) - \frac{M^2 R_{AdS}^4}{u_\Lambda^2} \right) e^{-2y} + \cdots \\ V(y \to -\infty) &\approx \left(\frac{1}{4}(p+1)(5p+9) - \frac{M^2 R_{AdS}^4}{(p+1)u_\Lambda^2} \right) e^y + \cdots \,. \end{aligned} \tag{3.9}$$

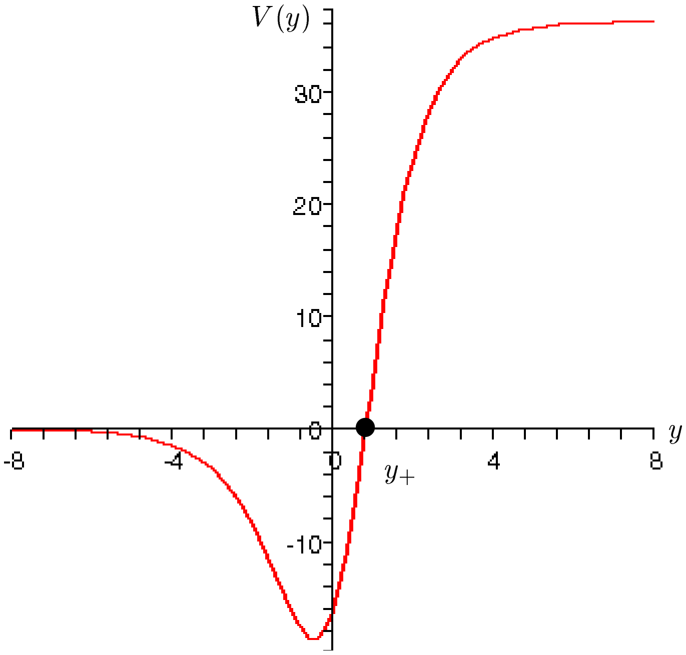

Figure 1: The effective potential (3.8) for $p = 4$ and $\frac{MR^2_{AdS}}{u_\Lambda} = 20$. The plot demonstrates that there are two classical turning points at $y = y_+$ and at $y = -\infty$.

It means that to the leading order in $\frac{MR^2_{AdS}}{u_\Lambda}$ the classical turning points are:

$$y_+ = \ln\left(\left(\frac{4}{(p+1)(5p+9)}\right)^{1/2}\frac{MR^2_{AdS}}{u_\Lambda}\right) \qquad \text{and} \qquad y_- = -\infty\,. \tag{3.10}$$

Note that the point y_- corresponds in terms of the original radial coordinate to $u = u_\Lambda$. In the WKB approximation the potential satisfies:

$$\left(k - \frac{1}{2}\right)\pi = \int_{y_-}^{y^+} \sqrt{-V(y)}dy\,, \tag{3.11}$$

where k is a positive integer. Using the method of [26] we expand the integral as a series in powers of $\frac{u_\Lambda}{MR^2_{AdS}}$ leaving only the terms appearing at $O(M)$ and $O(M^0)$. To leading order in M the integral (3.11) is approximated by:

$$\int_{-\infty}^{y^+} \sqrt{-V(y)}dy \approx \frac{MR^2_{AdS}}{u_\Lambda}\int_{-\infty}^{+\infty} \frac{e^y(1+e^y)^{\frac{p-3}{2}}}{((1+e^y)^{p+1}-1)^{1/2}}dy \approx \frac{MR^2_{AdS}}{u_\Lambda}\frac{\sqrt{\pi}\Gamma\left(\frac{1}{p+1}\right)}{(p+1)\Gamma\left(\frac{1}{2}+\frac{1}{p+1}\right)}\,. \tag{3.12}$$

There are two contributions to the next order term in the $1/M$ expansion. The first contribution comes from integrating to ∞ instead of y_+ in the leading order term. Therefore we should subtract from the result the following contribution:

$$\frac{MR^2_{AdS}}{u_\Lambda}\int_{y^+}^{\infty} \frac{e^y(1+e^y)^{\frac{p-3}{2}}}{((1+e^y)^{p+1}-1)^{1/2}}dy \approx \frac{1}{2}\left((p+1)(5p+9)\right)^{1/2}. \tag{3.13}$$

The second contribution comes from the integration near $y = y_+$:

$$\int^{y_+} \left(\sqrt{-V(y)} - \frac{MR^2_{AdS}}{u_\Lambda} \frac{e^y(1+e^y)^{\frac{p-3}{2}}}{((1+e^y)^{p+1}-1)^{1/2}} \right) dy \approx \frac{1}{2}\left((p+1)(5p+9)\right)^{1/2} \left(1 - \frac{\pi}{2}\right). \tag{3.14}$$

Finally, adding up and solving for M in terms of k and substituting $p = 4$ we obtain the masses of the spin-0 dilaton excitations:

$$M^2_{\mathbf{0}^{++},\phi} \approx \frac{39.66}{\beta^2} k(k+5.02) + O(k^0) \qquad \text{with } \beta = \frac{4\pi R^2_{AdS}}{5u_\Lambda}. \tag{3.15}$$

We can also find the spectrum using the "shooting technique". Solving (3.5) numerically and matching the boundary condition at $u = u_\Lambda$ and $u = \infty$ results in a discrete set of eigenvalues of M_k^2. Table 1 compares the WKB expression (3.15) and the numerical results. We see a nice agreement which improves for higher k.

k	WKB	Numerical
1	15.45	19.09
2	23.60	26.14
3	30.89	32.88
4	37.83	39.47
5	44.58	45.98
6	51.21	52.44

Table 1: Comparison of the 0^{++}-glueball masses $M_{0^{++},\phi}$ in units of β^{-1}. The WKB approximation is very close to the numerical results.

Next let us analyze the scalar glueballs related to the RR 1-form A_M directed completely along the compact θ-coordinate. We will consider the ansatz $A_\theta = a(u)e^{ikx}$ with all other components vanishing identically. It will be useful to re-write the 1-form equation of motion (3.4) as:

$$\partial_M \left(\sqrt{g} g^{NK} g^{ML} (\partial_K A_L - \partial_L A_K)\right) = 0. \tag{3.16}$$

For our ansatz the only non-trivial equation occurs for $N = \theta$. The 2-nd order differential equation for $a(u)$ reads:

$$\partial_u^2 a(u) + \frac{p}{u}\partial_u a(u) + M^2 R^4_{AdS} \frac{u^{p-3}}{u^{p+1} - u_\Lambda^{p+1}} a(u) = 0. \tag{3.17}$$

Following the same steps as in the dilaton case we end up with the following effective potential:

$$V(y) = \frac{1}{4} + \frac{p(p-2)e^{2y}}{4(1+e^y)^2} - \left(\frac{MR^2_{AdS}}{u_\Lambda}\right)^2 \frac{e^{2y}(1+e^y)^{p-3}}{(1+e^y)^{p+1}-1}. \tag{3.18}$$

The typical form of the potential is presented on figure 2. In this case the classical turning points are situated at:

$$y_+ \approx \ln\left(\frac{2MR^2_{AdS}}{(p-3)u_\Lambda}\right) \qquad \text{and} \qquad y_- \approx -2\ln\left(\frac{2MR^2_{AdS}}{\sqrt{p+1}u_\Lambda}\right). \tag{3.19}$$

Note that in this case the inner turning point is located away from the surface $u = u_\Lambda$. Finally, the spectrum of the glueballs related to the RR 1-form perturbation directed along the compact coordinate is:

$$M^2_{\mathbf{0}^{-+},A_\theta} \approx \frac{39.66}{\beta^2} k\left(k + \frac{3}{2}\right) + O(k^0). \tag{3.20}$$

We compare this result to the numerical solutions of (3.17) in table 2. Again the agreement is very close.

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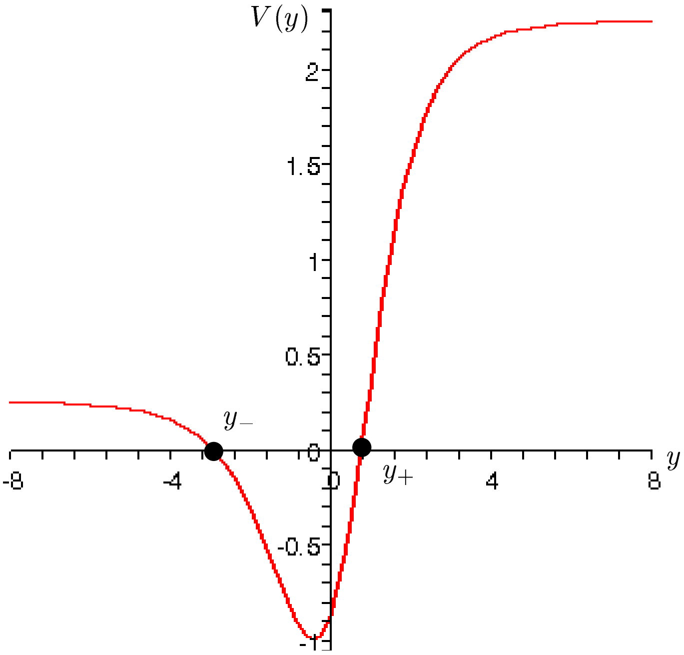

Figure 2: The effective potential (3.18) for $p = 4$ and $\frac{MR_{AdS}^2}{u_\Lambda} = 5$. There are two classical turning points at $y = y_+$ and at $y = y_-$.

k	WKB	Numerical
1	9.96	10.21
2	16.67	16.81
3	23.14	23.25
4	29.54	29.62

k	WKB	Numerical
1	7.72	7.44
2	14.08	13.96
3	20.41	20.32
4	26.72	26.65

Table 2: Comparison of the 0^{-+} glueball masses M_{0^{-+},A_θ} (left) and the 1^{++} glueball masses M_{1^{++},A_μ} (right) in units of β^{-1}. For 0^{-+} masses the WKB approximation is close to the numerical results for any k.

Next let us consider the RR 1-form with legs along the non-compact coordinates x_μ's. Now the ansatz is $A_\mu = v_\mu \alpha(u) e^{ikx}$ with $v \cdot k = 0$ and plugging this into (3.16) we get:

$$\partial_u^2 \alpha(u) + \frac{pu^{p+1} + u_\Lambda^{p+1}}{u\left(u^{p+1} - u_\Lambda^{p+1}\right)} \partial_u \alpha(u) + M^2 R_{AdS}^4 \frac{u^{p-3}}{u^{p+1} - u_\Lambda^{p+1}} \alpha(u) = 0\,. \tag{3.21}$$

In this case the effective potential is:

$$\begin{aligned} V(y) = {} & \frac{1}{4} + \frac{e^{2y}\left(p(p-2)(1+e^y)^{2(p+1)} - 2(p^2+2)(1+e^y)^{p+1} - 1\right)}{4(1+e^y)^2((1+e^y)^{p+1}-1)^2} - \\ & - \left(\frac{MR_{AdS}^2}{u_\Lambda}\right)^2 \frac{e^{2y}(1+e^y)^{p-3}}{(1+e^y)^{p+1}-1} \end{aligned} \tag{3.22}$$

and the classical turning points are:

$$y_+ \approx \ln\left(\frac{2MR^2_{AdS}}{(p-1)u_\Lambda}\right) \qquad \text{and} \qquad y_- \approx -\infty\,. \tag{3.23}$$

The WKB mass formula for the spectrum of the glueballs associated with RR 1-form directed along the non-compact coordinates is:

$$M^2_{\mathbf{1}^{++},A_\mu} \approx \frac{39.66}{\beta^2} k\left(k+\frac{1}{2}\right) + O(k^0)\,. \tag{3.24}$$

We compare this result to the numerical solutions of (3.21) in table 2. Again the agreement is very close.

3.2 The graviton spectra

The wave equations (3.2) for the metric fluctuations about the AdS_{p+1} black hole background have been analyzed by [24]. Here we will use these results for $p=4$. For different polarizations the linearized equation (3.2) reproduces a set of differential equations for spin-2, spin-1 and spin-0 glueballs. The spin-2 part corresponds to the graviton polarized in the direction parallel to the hyper-surface spanned by the world-volume coordinates x_μ's. The appropriate ansatz for the metric in this case is:

$$h_{ab} = \epsilon_{ab}\left(\frac{u}{R_{AdS}}\right)^2 H(u)\,, \tag{3.25}$$

where ϵ_{ab} is a constant traceless polarization tensor and the differential equation for the function $H(u)$ is:

$$\partial_u\left(\left(u^{p+2} - u_\Lambda^{p+1}u\right)\partial_u H(u)\right) - M^2 R^4_{AdS} u^{p-2} H(u) = 0\,. \tag{3.26}$$

The effective potential in derived from this equation is:

$$\begin{aligned} V(y) \;=\; & \frac{1}{4} + \frac{e^{2y}\left(p(p+2)(1+e^y)^{2(p+1)} - 2p(p+2)(1+e^y)^{p+1} - 1\right)}{4(1+e^y)^2((1+e^y)^{p+1}-1)^2} - \\ & - \left(\frac{MR^2_{AdS}}{u_\Lambda}\right)^2 \frac{e^{2y}(1+e^y)^{p-3}}{(1+e^y)^{p+1}-1}. \end{aligned} \tag{3.27}$$

It was pointed out in [24] that this effective potential is identical to the potential one obtains for a minimally coupled scalar ($\nabla^2\phi = 0$). Based on this observation the authors concluded that there is a degeneracy between the tensor and the scalar excitation. As we see, however, in the framework of non-critical supergravity the dilaton is not minimally coupled, but rather satisfies the equation (3.3), and therefore the effective potentials (3.8) and (3.27) are different.

Following the same steps as in the scalar glueball case we can find the WKB expression for the spectrum and compare it to the numerical results obtained directly from (3.26). The final result is:

$$M^2_{\mathbf{2}^{++},h_{\mu\nu}} \approx \frac{39.66}{\beta^2} k\left(k+\frac{3}{2}\right) + O(k^0)\,. \tag{3.28}$$

k	WKB	Numerical
1	11.35	12.57
2	18.36	19.43
3	24.99	25.99
4	31.49	32.44

Table 3: Comparison of the 1^{-+} glueball masses $M_{1^{-+},h_{\theta\mu}}$ in units of β^{-1}.

k	WKB	Numerical
1	9.96	6.34
2	16.67	15.58
3	23.14	22.43
4	29.54	29.01

Table 4: Comparison of the 0^{++} glueball masses $M_{0^{++},h_{\theta\theta}}$ in units of β^{-1}. The WKB approximation is close to the numerical results only for $k > 1$.

Surprisingly this expression is identical to the result (3.20) for the 0^{-+} glueballs in the previous subsection. Moreover, this degeneracy holds also beyond the WKB approximation, since the numerical calculations also produce the same spectrum. This is quite unexpected, since there is no redefinition of the function $H(u)$ or/and of the radial coordinate u, that brings the differential equation (3.26) to the form (3.17)! We will return to this result in the end of the section, while comparing our results to the lattice calculations.

Finally, let us quote the results for the spin-1 and spin-0 modes. The solution that appears as vector in the gauge theory is given by the ansatz (3.17) with the polarization tensor satisfying:

$$\epsilon_{\theta\mu} = v_\mu \,, \qquad \text{where} \qquad k \cdot v = 0 \quad \text{and} \quad v^2 = 1 \,. \tag{3.29}$$

The ansatz is consistent with the equation of motion provided that $H(u)$ satisfies:

$$\partial_u^2 H(u) + \frac{(p+2)}{u}\partial_u H(u) + \frac{M^2 R_{AdS}^4 u^{p-3}}{\left(u^{p+1} - u_\Lambda^{p+1}\right)} H(u) = 0 \,. \tag{3.30}$$

The effective potential in this case is given by:

$$V(y) = \frac{1}{4} + \frac{p(p+2)e^{2y}}{4(1+e^y)^2} - \left(\frac{M R_{AdS}^2}{u_\Lambda}\right)^2 \frac{e^{2y}(1+e^y)^{p-3}}{(1+e^y)^{p+1} - 1} \,. \tag{3.31}$$

For $p = 4$ the WKB approximation to the 1^{-+}-glueball spectrum is:

$$M^2_{\mathbf{1}^{-+},h_{\theta\nu}} \approx \frac{39.66}{\beta^2} k \left(k + \frac{9}{4}\right) + O(k^0) \,. \tag{3.32}$$

Table 3 compares the WKB expression to the numerical results.

The scalar perturbation of the metric leads to a complicated set of differential equation and here we will only quote the result of [24] for the WKB formula:

$$M^2_{\mathbf{0}^{++},h_{\theta\theta}} \approx \frac{39.66}{\beta^2} k \left(k + \frac{3}{2}\right) + O(k^0) \,. \tag{3.33}$$

Remarkably this expression reproduces the result (3.28) for the 2^{++}-glueballs. This degeneracy, however, does not hold beyond the WKB approximation as one can see comparing the spectrum of the 2^{++} and 0^{-+}-glueballs in table 2 and the results in table 4, where we present the numerical results for the 0^{++}-glueballs.

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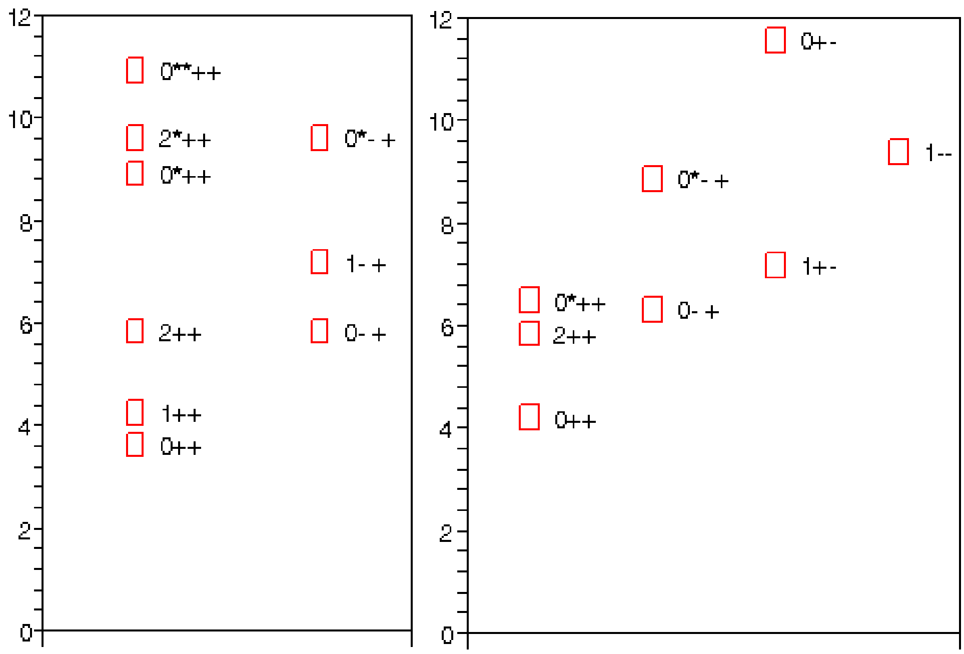

Figure 3: The AdS glueball spectrum for YM_4 computed in the framework of non-critical supergravity (left) and the corresponding lattice results (right). The AdS scale is adjusted to set the lowest 2^{++} state to the the lattice result in units of the hadronic scale $1/r_0 = 410\text{MeV}$.

3.3 Comparison

In figure 3 we compare the glueball spectrum for YM_4 in strong coupling calculated above with the lattice spectrum [29] for pure SU(3) YM_4. In figure 4 we collect the results one obtains using the critical supergravity approach. Few remarks are in order:

- Although we did not succeed to reproduce accurate mass ratios for all glueball states, we see that there is a remarkable similarity between our strong coupling spectrum and the lattice results. For example, from the lattice computations one has $M(0^{-+})/M(0^{++}) \approx 1.504$, while our calculation gave $M(0^{-+})/M(0^{++}) \approx 1.610$.

- As in the critical supergravity calculations the lowest 0^{++}-glueball state comes from the scalar component of the metric, and not from the dilaton.

- We saw above that the masses of the 2^{++} and the 0^{-+}-glueball are identical even beyond the WKB approximation. Remarkably, in the lattice YM spectrum the lowest masses of these states are quite close, more precisely $M(2^{++})/M(0^{-+}) \approx 1.082$. In the critical supergravity computation the ratio is $M(2^{++})/M(0^{-+}) \approx 1.203$.

- In Witten's setup [22] one starts from the $AdS_7 \times S^4$ background and than introduces two circles $S^1 \times S^1$ with the anti-periodic boundary conditions on one of them and taking the radii to zero one reduces the world volume to four dimensions. In this limit M theory reduces to type-IIA string theory on the non-conformal D_4 background with infinite temperature. Let us denote the coordinates of the AdS_7 black hole metric

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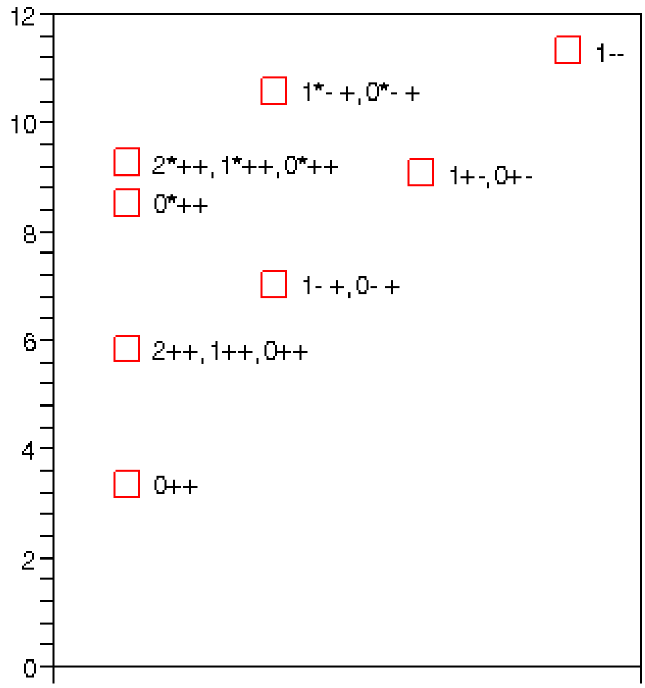

Figure 4: The AdS glueball spectrum for YM_4 computed in the near extremal $AdS_7 \times S^4$ background with further reduction to the type-IIA SUGRA solution. The AdS scale is adjusted to set the lowest 2^{++} state to the the lattice result in units of the hadronic scale $1/r_0 = 410\text{MeV}$.

by θ, $x_{i=1,2,3,4}$, x_{11} and u. Here θ and x_{11} are the two compact directions and u is the radial coordinates. The reduction to the type-IIA background corresponds to the compactification on x_{11}. The graviton polarization tensor has $(5 \times 6)/2 - 1 = 14$ independent components. It decomposes into 9-dimensional tensor, 4-dimensional vector and 1-dimensional scalar irreducible representation. The graviton equation of motion, therefore, leads to three distinct wave equations as we saw in the previous subsection. This immediately implies that after dimensional reduction to $10d$ we obtain a *degenerate* spectrum. In particular, the tensor wave equation (3.26) for $p = 5$ will lead to the 2^{++}, 1^{++} and 0^{++} degenerate glueball spectrum in $d = 4$. Similarly the vector equation (3.30) for $p = 5$ corresponds to the 1^{++} and 0^{++} glueballs. There is no degeneracy between these states in our approach, since we have only *one* compact coordinate. Moreover, there is a scalar field h^{α}_{α} coming from the trace of the metric on the S^4, which is related to the 0^{++}-state in the gauge theory. For this field the critical supergravity prediction is $m_{0^{++},h^{\alpha}_{\alpha}}/m_{2^{++}} \approx 2.28$. In our model this field do not appear since there is no compact transversal space.

- For large k the WKB expressions (3.15), (3.20), (3.28), (3.32), (3.33) for the glueball masses reduce to
$$M \approx \frac{C_p}{\beta} k\,, \tag{3.34}$$
where C_p is a constant depending on p. In the critical supergravity calculation ($p = 5$) one has $C_5 \approx 5.42$ and in our case ($p = 4$) we get $C_4 \approx 6.30$. It means that we obtain

different asymptotic behaviors in our non-critical description and Witten's model. Unfortunately, we cannot compare this result to the lattice YM, since it provides only masses of the lowest states.

4. The Wilson loop

The stringy description of the Wilson loop [30, 31] is in terms of a NG string whose end-points are "nailed" at two points on the boundary of the AdS black hole space-time, namely at $u = \infty, x = \pm L/2$, where x denotes one of the 3d space directions. In [32] the classical energy of the Wilson loop associated with a background with a general dependence on the radial direction was written down. In particular it was proved that a sufficient condition for an area law behavior is that:

$$g_{uu} g_{00}(u_d) \to \infty \qquad \text{and} \qquad g_{00}(u_d) > 0\,, \tag{4.1}$$

where u_d is a particular point along the u direction. In this case the string tension is given by $T_s = g_{00}(u_d)$.

It is very easy to verify that our background metric (2.1) obeys this condition for $u_d = u_\Lambda$ and hence the potential of a quark anti-quark pair of the dual gauge theory is indeed linear in the separation distance L. In fact using the results of [32], we can write down the full expression for the classical energy of the Wilson line. It takes for our case the following form:

$$E = \frac{1}{2\pi}\left(\frac{u_\Lambda}{R_{AdS}}\right)^2 \cdot L - 2\kappa + \mathcal{O}\left((\log L)^\gamma e^{-\alpha L}\right) \tag{4.2}$$

where $\alpha = \sqrt{5}\frac{u_\Lambda}{R_{AdS}^2}$, γ is a positive constant and the constant κ is given by:

$$\kappa = \frac{1}{2\pi}\int_{u_\Lambda}^{\infty} du \left(\left(1 - \left(\frac{u_\Lambda}{u}\right)^5\right)^{-1/2} - 1\right) \approx 0.309\frac{u_\Lambda}{2\pi}\,. \tag{4.3}$$

4.1 The Luscher term

The Luscher term is the sum of the contributions of quantum fluctuations that adds to the classical quark anti-quark potential. In a superstring it incorporates both the bosonic as well as the fermionic fluctuations. In the present case of the non-critical string, we lack a formulation of the fermionic part of the action and in fact it is plausible that even prior to invoking the anti-periodic boundary condition along the thermal circle, the model is not space-time supersymmetric. Hence, we discuss here only the bosonic quantum fluctuations. However, it might be that in spite of this ignorance we can predict the form of the full contribution of the quantum fluctuations. Recall that in the critical case, due to the coupling to the RR fields, all the fermionic modes are massive so that in that case the contributions of the massless modes are coming only from the bosonic sector. Since in our non-critical model we also have RR fields it is quite plausible that a similar mass generation for the fermions will take place.

The fluctuations along the bosonic directions fall into two classes: massless modes and massive modes. The latter contribute to the quark anti-quark potential a Yukawa like term $\sim e^{-mL}$, where m is the mass of the mode. Since in computing Wilson loops we take $L \to \infty$, the contribution of massive modes is negligible. The contribution of a massless mode has the form $-\frac{1}{24}\frac{\pi}{L}$. Thus what is left to be determined is the number of massless modes. It turns out that this issue, which is very crucial when comparing to the lattice result, is a subtle one and there are contradicting claims about it in the literature. In [33, 34] in the context of the critical theory it was found that there are two massive bosonic modes. Applying this result to our case it will imply that there are altogether $6-2-2=2$ massless modes and hence $\Delta E_B \sim -\frac{2}{24}\frac{\pi}{L}$. It is very tempting to adopt this result since it agrees with the current value found in lattice calculations. However, following [35] we claim that in fact the number of massive modes is one and hence we end up with a Luscher term:

$$\Delta E_B = -\frac{3}{24}\frac{\pi}{L} \tag{4.4}$$

whereas in the critical case the result was $\Delta E_B = -\frac{7}{24}\frac{\pi}{L}$. Obviously even without using the results of [33, 34], the outcome of the non-critical string model is closer to the value measured in the lattice calculations than the result of the critical string model.

Since for the comparison with lattice simulations, the difference between the result of [33, 34] and our claim is important, let us briefly comment about the source of the discrepancy following the derivation of [35]. The idea there is to fix the gauge of the NG action by $\tau = t, \sigma = u_{cl}$ so that the fluctuation in the (x, u) plane is along the normal to the classical configuration. It was shown that choosing this gauge avoids potential problems of the $\sigma = u$ and $\sigma = x$ gauges. It was further shown that one of the modes that was found in [33] to be massive must be massless since it corresponds to a Goldstone boson associated with a breaking of a rotation invariance [35].

4.2 The correlator of two Wilson loops

The correlator of two Wilson loops in the context of the $AdS_5 \times S^5$ model was discussed in [36, 37]. It was shown that when the separation of the two loops in AdS_5 is of the order of their size, there is a solution of the string equation of motion that describes a connected surface ending on the two loops. As the separation increases the tube shrinks, and becomes unstable. At large distances the correlation is due to the exchange of supergravity modes in the bulk between the world-sheets of the loops. The long distance correlator was calculated in [38]. An analysis of the long distance correlator for the near extremal $AdS_5 \times S^5$ model in the limit of large temperature that corresponds to the low energy effective action of the pure three dimensional YM theory was performed in [19]. It was shown that the correlator was dominated by an exchange of a KK "scalarball" mode which is lighter than any of the glueball modes.

Let us now address the correlator for our non-critical near extremal AdS_6 model. It is plausible that the transition between a connecting world-sheet into an exchange of a SUGRA modes will take place here also. For separation distances much greater than the distance between the endpoints of each string, the correlator is given by:

$$\log\left[\frac{\langle W(0)W(R)\rangle}{\langle W(0)\rangle\,\langle W(R)\rangle}\right]=\sum_{i,k}\int\mathcal{A}_1\int\mathcal{A}_2\;f_1^{i,k}\;f_2^{i,k}\;G^{i,k}\,, \tag{4.5}$$

where R is the distance between the two loops, $\mathcal{A}_i$ are the areas of the two loops, k is the mode number and $f_1^{i,k}$, $f_2^{i,k}$ are the couplings of the field i to the world-sheet and $G^{i,k}$ is the propagator. There are two clear differences between this expression and the corresponding one in the critical case: (i) The sum of the modes here does not include any of the KK modes associated with the S^5 and hence only glueballs (and modes associated with the thermal S^1). (ii) For each of the modes like the dilaton, the graviton etc. we do not sum over all the spherical harmonics associated with the S^5. The dilaton coupling follows simply from the relation between the Einstein and string frame metrics, $g_s = g_E e^{\phi}$. Therefore the one dilaton coupling is 1. The coupling to the other modes are in general u dependent [19].

There are at least three types of correlators that one can study: (i) Correlators of circular loops located on planes in the space spanned by the x^i coordinates. (ii) Correlators of infinite strips along (t, x^i) planes (iii) Wilson loops along (θ, x^i). The first two types where discussed first for the $AdS_5 \times S^5$ string in [38] and in [19] the three types were discussed in the near extremal case which is dual to the low energy regime of $3d$ pure YM theory (contaminated with KK modes). In a similar way the correlator of type (ii) in our non-critical model should correspond to the potential between two external mesons of $4d$ pure YM theory. The result we find for this potential is:

$$V_{mm}\sim\frac{1}{T}\log\left[\frac{\langle W(0)W(R)\rangle}{\langle W(0)\rangle\,\langle W(R)\rangle}\right]\sim\frac{1}{N^2}K_0(M_{\varphi_{00}}R)\,, \tag{4.6}$$

where T is the temporal length of each strip, K_0 is a modified Bessel function and φ_{00} is the lightest scalar glueball mode. Based on the analysis of section 3, this mode will be the 0^{++} that associates with the graviton. Note that the amplitude scales like $\frac{1}{N^2}$. This is also the behavior in the critical case. In the latter case the amplitude is also proportional to $g_{YM}^2 N$ which relates to the radius of the AdS_5 and hence does not show up in our formulation.

5. The 't Hooft loop

It is well known that in confining gauge theories a monopole anti-monopole are screened from each other. The corresponding potential is determined via the 't Hooft loop in a similar way that the potential between a quark and anti-quark is determined from the Wilson loop. The stringy description of a monopole is a $D2$ brane that ends on a D4 brane so that the 't Hooft loop is described by a $D2$ brane that is attached to the boundary of the background in two points. In the context of critical string theories the 't Hooft loop was determined in [39] for the case of the near extremal D4 brane background, and it was shown that indeed in that case the system of a monopole and anti-monopole was energetically favorable in comparison with the bound state system, which implies that the monopole and anti-monopole are indeed screened from each other.

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Let us perform a similar calculation for our setup. We take the world-volume of the $D2$ brane to be along the directions t, x and θ, namely it wraps the S^1. We will assume that the radial coordinate u along the $D2$ brane depends only on x. The action of the $D2$ brane takes the form:

$$S = \frac{1}{(2\pi\alpha')^{3/2}} \int d\tau d\sigma_1 d\sigma_2 e^{-\phi_0} \sqrt{\det h_{\rm ind}} = \beta \int_{-L/2}^{L/2} dx \frac{u}{R_{AdS}} \sqrt{u'^2 + \left(\frac{u}{R_{AdS}}\right)^4 f(u)}\,, \tag{5.1}$$

where $f(u) = 1 - (u_\Lambda/u)^5$ is the thermal factor and L is the separation distance between the pair since the action does not depend on x explicitly the solution of the equation of motion satisfies:

$$\left(\frac{u}{R_{AdS}}\right)^5 f(u) \left(u'^2 + \left(\frac{u}{R_{AdS}}\right)^4 f(u)\right)^{-1/2} = \text{const}\,. \tag{5.2}$$

Defining $u_{\rm min}$ to be the minimal value $u(x)$ we arrive at the following expression for the separation distance L:

$$L = \int dx = 2\int_{u_\Lambda}^{\infty} \frac{du}{u'} = 2\frac{R_{Ads}^2}{u_{\rm min}} \epsilon^{1/2} \int_1^\infty dy \frac{y}{\sqrt{(y^5 - 1 + \epsilon)(y^5 - \epsilon y - 1 + \epsilon)}}\,, \tag{5.3}$$

where $\epsilon \equiv f(u_{\rm min})$. For $u_{\rm min} \to u_\Lambda$ we have $\epsilon \to 0$ and $L \to \infty$. To compute the binding energy we have to subtract the masses of the free monopole and anti-monopole, namely the energy of two parallel $D2$ branes stretched from the boundary to the horizon. The final result is:

$$\Delta E \sim \beta \frac{u_{\rm min}^2}{u_\Lambda} \left[\int_1^\infty dy\, y^2 \left(\sqrt{\frac{y^5 - 1 + \epsilon}{y^5 - \epsilon y - 1 + \epsilon}} - 1\right) - \frac{1}{3}\left(1 - \left(\frac{u_\Lambda}{u_{\rm min}}\right)^3\right)\right]. \tag{5.4}$$

For $L\beta^{-1} \gg 1$ ($\epsilon \approx 0$) the energy is positive which means that the zero-energy configuration of two parallel $D2$ branes ending on the horizon is energetically favorable. We conclude therefore that in the "YM region" there is no force between the monopole and the anti-monopole and there is a screening of the magnetic charge.

Finally we should note that though our final conclusion matches the results of [39], the explicit expression for the energy (5.4) differs from the expression derived in [39].

6. Closed spinning strings in the AdS_{p+1} black hole background

The glueball spectrum that was extracted from the supergravity in section 2 is obviously limited to states of spin not higher than two. To study the spectrum of glueballs of higher spin one has to consider stringy configurations rather than supergravity modes. The string configurations which are dual to glueballs of higher spin are spinning folded closed strings [40]. In particular we would like to investigate the possibility that the high spin glueballs furnish a close string Regge trajectory. Our task is to check whether the non-critical strings associated with the AdS_6 black hole admit classical spinning configurations, compute the relation between the angular momentum and energy of such configurations and incorporate quantum fluctuations. This type of analysis was done previously in the context of AdS black hole [41], in the framework of the KS and MN models in [42] and recently for the critical near extremal D4 brane in [34].

Let us analyze now what are the condition of having classical spinning string solutions of the equations of motion. Since the background depends only on the radial coordinate u, the equation of motion with respect to this coordinate plays a special role. Suppose now that we perform a coordinate transformation $u \to \rho(u)$, then the equation of motion with respect to ρ in the Polyakov formulation reads:

$$-2\partial_\alpha(g_{\rho\rho}\partial^\alpha\rho) + \frac{dg_{\rho\rho}}{d\rho} - \partial_\alpha\rho\partial^\alpha\rho\frac{dg_{00}}{d\rho}\partial_\alpha t\partial^\alpha t - + \frac{dg_{ii}}{d\rho}\partial_\alpha x^i\partial^\alpha x^i + \frac{d - g_{\theta\theta}}{d\rho}\partial_\alpha\theta\partial^\alpha\theta = 0\,. \quad (6.1)$$

It is trivial to check that a spinning string of the form:

$$X^0 = e\tau \qquad X^1 = e\cos\tau\sin\sigma \qquad X^2 = e\sin\tau\sin\sigma$$
$$\theta = \text{const} \qquad \rho = \text{const}\,, \quad (6.2)$$

solves the other equations of motion and that for such a configuration the first two terms of (6.1) and the last vanish. Since the configuration has non trivial X^0 and X^i, (6.1) is obeyed if at a certain value of $\rho = \rho_0$ $\frac{dg_{00}}{d\rho}|_{\rho=\rho_0} = \frac{dg_{ii}}{d\rho}|_{\rho=\rho_0} = 0$. It is natural to make a coordinate transformation such that around the horizon:

$$g_{uu}du^2 = d\rho^2 \to \qquad \frac{d\rho}{du} = \frac{R_{AdS}}{u}f^{1/2}(u)\,. \quad (6.3)$$

In the new coordinates u_Λ, namely the horizon, is mapped into $\rho = 0$ as can be seen from

$$\rho \approx \frac{2R_{AdS}}{(5u_\Lambda)^{1/2}}(u - u_\Lambda)^{1/2} = \frac{\beta}{2\pi}(u - u_\Lambda)^{1/2}\,, \quad (6.4)$$

and for the metric:

$$-g_{00} = g_{ii} \approx \left(\frac{u_\Lambda}{R_{AdS}}\right)^2 + \frac{5u_\Lambda^2}{2R_{AdS}^4}\cdot\rho^2 + \cdots\,. \quad (6.5)$$

It implies that:

$$g_{00}|_{\rho=0,u=u_\Lambda} \neq 0 \qquad \text{and} \qquad \partial_\rho g_{00}|_{\rho=0,u=u_\Lambda} = 0\,. \quad (6.6)$$

which means that indeed for a spinning configuration at $\rho = 0$ the equation of motion (6.1) is obeyed. In fact this is precisely one of the two possible sufficient conditions to have an area law Wilson loop [32]. Moreover, it implies that the spinning string stretches along the horizon which is often referred to as the “wall” or the “ end of the world floor” exactly as the static configuration of the string that corresponds to the Wilson loop does. This relation between the condition for confinement and for spinning string configurations and the fact that the string, like the Wilson loop string, is along the wall, was observed also in [42] for folded close string in the KS and MN models and for open strings in the near extremal D4 brane model in [42]. It is very plausible that this relation is universal and applies to any SUGRA dual of a confining gauge theory.

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It is also easy to check that the classical configuration (6.2) at the wall $\rho = 0$ obeys on top of the equations of motion also the Virasoro constraint. For this solution the classical energy of the string configuration is:

$$E = \frac{e}{2\pi\alpha'}\int g_{00}|_{\rho=0} d\sigma = 2\pi T_{\rm s} e g_{00}|_{\rho=0} \,. \tag{6.7}$$

Similarly the classical angular momentum is:

$$J = \pi T_{\rm s} e^2 g_{ii}|_{\rho=0} \,. \tag{6.8}$$

Recalling that $g_{00}(0) = g_{ii}(0) \neq 0$ we end up with the following result:

$$J = \frac{1}{2}\alpha'_{\rm eff}E^2 = \frac{1}{2}\alpha'_{\rm eff}t \qquad \text{where } \alpha'_{\rm eff} = \frac{\alpha'}{g_{00}(0)} = \frac{\alpha'}{\alpha'\left(\frac{u_\Lambda}{R_{AdS}}\right)^2} = \frac{15/2}{u_\Lambda^2} \,. \tag{6.9}$$

6.1 Quantum corrections — deviations from linearity

It is well known that the basic linear relation between the angular momentum and $E^2 \equiv t$ receives corrections. The "famous" correction is the intercept α_0 which is a constant t independent so that once included the trajectory reads $\frac{1}{2}\alpha'_{\rm eff}t + \alpha_0$. In the string derivation of the Regge trajectory the intercept is a result of the quantum correction and hence it is intimately related to the Luscher term. For instance in the bosonic string in d dimensions the intercept is $(d-2)\frac{\pi}{24}$. This result is achieved by adding quadratic fluctuations to the classical configurations and "measuring" the impact of these fluctuations on J and E. It turns out that in the canonical quantization of the Polyakov formulation, using the Virasoro constraint, one finds that [43]:

$$e(E - \bar{E}) = J - \bar{J} + \int d\sigma \mathcal{H}(\delta x^i) \,, \tag{6.10}$$

where $\bar{E}$ and $\bar{J}$ are the classical values of the energy and angular momentum and $\mathcal{H}(\delta x^i)$ is the world-sheet hamiltonian expressed in terms of the fluctuating fields. Using this procedure, as well as a path integral calculation, the contributions of the quantum fluctuations were computed in the KS and MN models [42] and in the near near extremal $D4$ model [34]. From the result of the latter one can be easily extract the form of the result also for our non-critical model. There are two apparent differences, though: (i) The number of physical bosonic directions which is here 4 rather 8, (ii) The fact that we do not have a formulation for the fermionic fluctuations in our model. Hence, we can only determine the contribution of the bosonic fluctuations to the non linearities of the trajectory. The bosonic modes include 3 massless modes and one massive mode. The latter has a σ-dependent mass. Its contribution was calculated in [34, 42]. Collecting all the contributions we get:

$$J = \frac{1}{2}\alpha'_{\rm eff}(E - z_0)^2 - \frac{3}{24}\pi + \Delta_{\rm f} \,, \tag{6.11}$$

where z_0 is proportional to u_Λ and $\Delta_{\rm f}$ will be the contribution of the massless and massive fermionic modes. We thus see that there is a non trivial bosonic intercept $\alpha_0 = \frac{1}{2}\alpha'_{\rm eff}z_0^2 - \frac{3}{24}\pi$, but in addition there is also a term linear in E. A difference between the result in the critical model [34] and the non-critical one is in the proportionally factor between z_0 and u_Λ.

Acknowledgments

We would like to thank Ido Adam, Yaron Oz and Ofer Aharony for fruitful discussions. J.S. would like to thank Leo Pando Zayas and Diana Vaman. He also wants to thank the Kavli Institute for Theoretical Physics at UC, Santa Barbara and the Department of Physics of the University of Texas at Austin, where part of this work was done. This work was supported in part by the German-Israeli Foundation for Scientific Research and by the Israel Science Foundation.

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